Software for impedance matching with lumped elements

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Some theory

Let us consider a load whose complex impedance $\underline{Z}_{\mathrm{Load}} = R_{\mathrm{Load}} + \mathrm{j}\, X_{\mathrm{Load}}$ is known, where $R_{\mathrm{Load}} > 0$ is the real part of the load impedance (the resistance), X_{Load} is the imaginary part of the impedance (the reactance), and $\mathrm{j} = \sqrt{-1}$ is the imaginary unit. The corresponding load complex admittance is $\underline{Y}_{\mathrm{Load}} = \frac{1}{\underline{Z}_{\mathrm{Load}}} = G_{\mathrm{Load}} + \mathrm{j}\, B_{\mathrm{Load}}$, where $G_{\mathrm{Load}} > 0$ is the real part of the admittance (the conductance) and B_{Load} is the imaginary part of the admittance (the susceptance). The operating frequency is f ($f > 0$). (The corresponding angular frequency is $\omega = 2\pi f$.)

Our task is to match the load to a given resistance[1] Z_0 ($Z_0 > 0$) using a linear, passive, lossless, reciprocal matching network (Fig. 1)[2]. In other words, the impedance looking into the matching network should be Z_0.

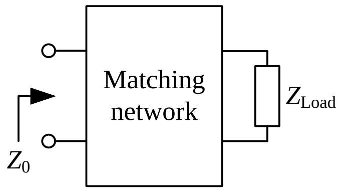


Fig. 1. Matching a load to a resistance using a lossless network.

We assume that the matching network consists of two lumped, reactive (lossless) elements. In that case, the matching network can have the two basic configurations (topologies) shown in Fig. 2, which are often referred to as L-networks. (In special cases, the matching network can be simpler, as discussed later.) In both configurations, we characterize the series element by its reactance (X) and the shunt element by its susceptance (B). Depending on the values of $\underline{Z}_{\mathrm{Load}}$ and Z_0, the lumped elements can be capacitors or inductors, so that we can have two capacitors, or two inductors, or one capacitor and one inductor.

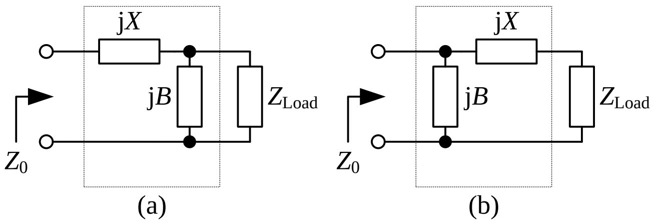
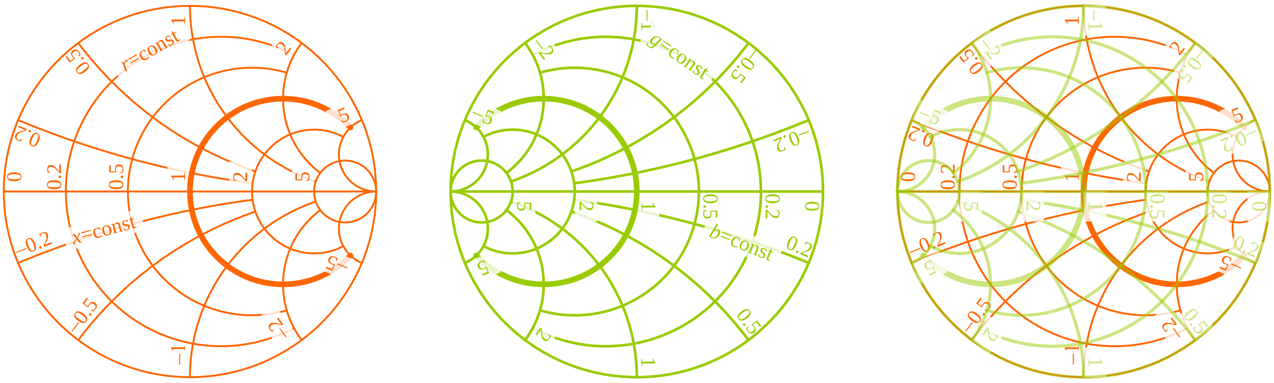


Fig. 2. Two basic configurations of the matching network that comprises two reactive elements.

In order to discuss the applicable configurations and the kinds of the reactive elements, we consider the Smith chart[3]. Fig. 3 shows the Smith chart with (a) the impedance grid, (b) the admittance grid, and (c) the two grids overlapped. The impedance and admittance are assumed to be normalized to a reference (nominal) impedance (or, equivalently a nominal admittance). We assume here this impedance to be Z_0.

[1] In practice, Z_0 is usually the characteristic impedance of a transmission line to which the load is to be connected. In the context of the scattering parameters (s-parameters), Z_0 is the nominal impedance.
[2] D. M. Pozar, *Microwave Engineering*, Fourth Edition, J. Wiley & Sons, 2012, Chapter 5, Section 5.1, pp. 228–234.
[3] We shall avoid formulas and derivations.



(a) (b) (c)

Fig. 3. Smith chart with (a) the impedance grid, (b) the admittance grid, and (c) the two grids overlapped.

Next, we draw in the Smith chart, shown in Fig. 3(c), the point that corresponds to the normalized load impedance, $\underline{z}_{\text{Load}} = \dfrac{\underline{Z}_{\text{Load}}}{Z_0} = r_{\text{Load}} + \mathrm{j}\,x_{\text{Load}}$, or, equivalently, that corresponds to the normalized load admittance, $\underline{y}_{\text{Load}} = \dfrac{1}{\underline{z}_{\text{Load}}} = g_{\text{Load}} + \mathrm{j}\,b_{\text{Load}}$. The number of possible solutions for the matching network can be zero, one, two, or four, depending on zone in Fig. 4 in which the point $\underline{z}_{\text{Load}}$ is located, according to the following list.

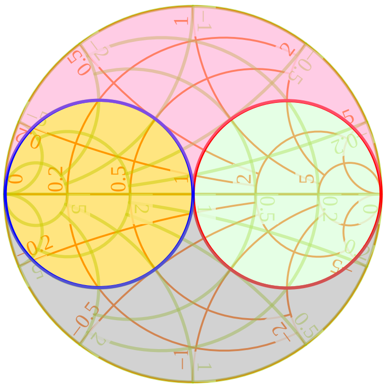


Fig. 4. Smith chart of Fig. 3(c) with zones. Please see the text for further explanations.

a. If $\underline{z}_{\text{Load}}$ is in the center of the Smith chart, i.e., if $r_{\text{Load}} = 1$ and $x_{\text{Load}} = 0$ (equivalently, if $g_{\text{Load}} = 1$ and $b_{\text{Load}} = 0$), the load is automatically matched, so that the matching network is not required. This is the **only solution**.
b. If $\underline{z}_{\text{Load}}$ is on the circle that bounds the Smith chart (the outermost circle), i.e., if the load impedance is purely imaginary (equivalently, if the load admittance is purely imaginary), there is **no solution**. However, this case is automatically excluded by the condition $R_{\text{Load}} > 0$, i.e., $r_{\text{Load}} > 0$ (or, equivalently, $G_{\text{Load}} > 0$, i.e., $g_{\text{Load}} > 0$).
c. If $\underline{z}_{\text{Load}}$ is on the red circle (i.e., if $r_{\text{Load}} = 1$), excluding the points that correspond to the cases (a) and (b), there are **two solutions**. In the first solution, the L-network is incomplete because it comprises only one reactive element connected in series with the load. The configuration of the second solution is shown in Fig. 2(a).
d. If $\underline{z}_{\text{Load}}$ is on the blue circle (i.e., if $g_{\text{Load}} = 1$), excluding the points that correspond to the cases (a) and (b), there are **two solutions**. In the first solution, the L-network is incomplete because it comprises only one reactive element connected in parallel with the load. The configuration of the second solution is shown in Fig. 2(b).
e. If $\underline{z}_{\text{Load}}$ is within the red circle, i.e., in the light green area, excluding the red circle itself (i.e., if $r_{\text{Load}} > 1$), the configuration shown in Fig. 2(a) is applicable. There exist **two solutions**, in which one reactive element is a capacitor and the other reactive element is an inductor.

f. If $\underline{z}_{\text{Load}}$ is within the blue circle, i.e., in the golden area, excluding the blue circle itself (i.e., if $g_{\text{Load}} > 1$), the configuration shown in Fig. 2(b) is applicable. There exist **two solutions**, in which one reactive element is a capacitor and the other reactive element is an inductor.

g. If $\underline{z}_{\text{Load}}$ is in the rose area, excluding its boundaries (i.e., if simultaneously $r_{\text{Load}} < 1$, $g_{\text{Load}} < 1$, and $x_{\text{Load}} > 0$), both topologies shown in Fig. 2 are applicable. There exist two distinct solutions for each topology, yielding a total of **four solutions**[4]. Two solutions comprise two capacitors and two solutions comprise one capacitor and one inductor.

h. If $\underline{z}_{\text{Load}}$ is in the gray area, excluding its boundaries (i.e., if simultaneously $r_{\text{Load}} < 1$, $g_{\text{Load}} < 1$, and $x_{\text{Load}} < 0$), both topologies shown in Fig. 2 are applicable. There exist two distinct solutions for each topology, yielding a total of **four solutions**. Two solutions comprise two inductors and two solutions comprise one capacitor and one inductor.

Examples

Example 1: $\underline{Z}_{\text{Load}} = (60 + \text{j}\,20)\,\Omega$ (i.e., $R_{\text{Load}} = 60\,\Omega$ and $X_{\text{Load}} = 20\,\Omega$), $Z_0 = 50\,\Omega$, and $f = 1\,\text{GHz}$. The point $\underline{z}_{\text{Load}}$ is in the light green area in Fig. 4 (case (e)). There exist two solutions, shown in Fig. 5.

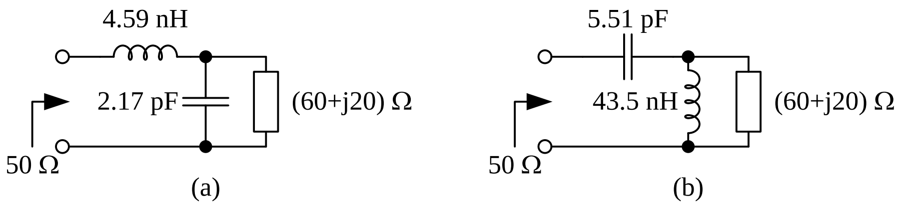


Fig. 5. Matching networks for Example 1.

Example 2: $\underline{Y}_{\text{Load}} = (40 + \text{j}\,20)\,\text{mS}$ (i.e., $\underline{Z}_{\text{Load}} = (20 - \text{j}10)\,\Omega$, $R_{\text{Load}} = 20\,\Omega$ and $X_{\text{Load}} = -10\,\Omega$), $Z_0 = 50\,\Omega$, and $f = 500\,\text{MHz}$. The point $\underline{z}_{\text{Load}}$ is in the golden area in Fig. 4 (case (f)). There exist two solutions, shown in Fig. 6.

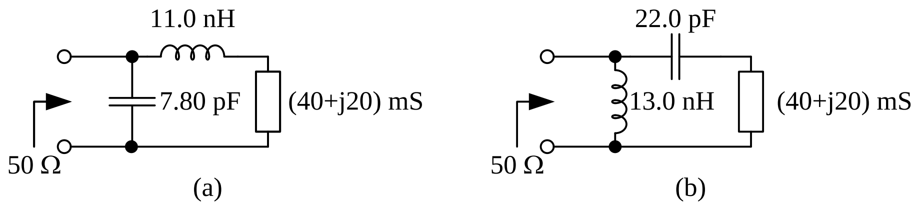


Fig. 6. Matching networks for Example 2.

Example 3: $\underline{Z}_{\text{Load}} = (50 + \text{j}100)\,\Omega$, $Z_0 = 100\,\Omega$, and $f = 2.45\,\text{GHz}$. The point $\underline{z}_{\text{Load}}$ is in the rose area in Fig. 4 (case (g)). There exist four solutions, shown in Fig. 7.

[4] The existence of a total of four solutions in the cases (g) and (h) is often overlooked.

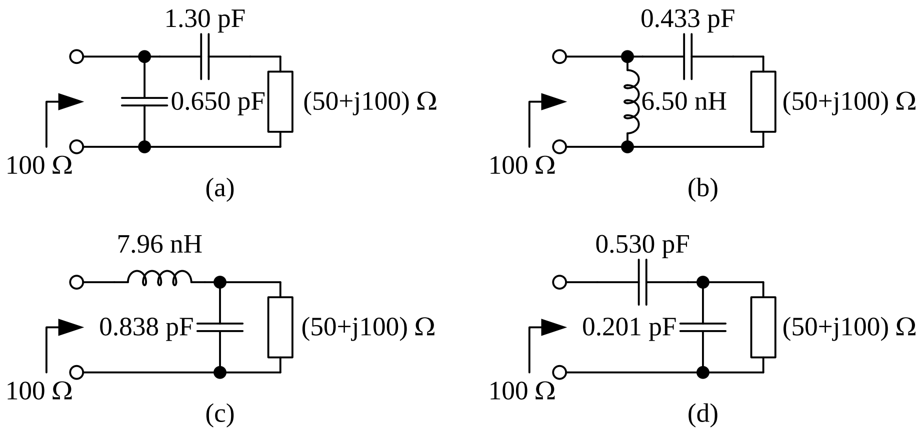


Fig. 7. Matching networks for Example 3.

Software

The following software is included in the package:

- matching_network.cpp – C++ source code;
- matching_network.exe – 64-bit executable (compiled matching_network.cpp), compatible with Windows 7 and later;
- matching_network.m – Matlab/Octave source code;
- matching_network.py – Python source code, compatible with Python 3.10 and later.

The input and output data are in the basic SI units.

For each program, the input data are R_{Load} (in Ω), X_{Load} (in Ω), Z_0 (in Ω), and f (in Hz). The input data are supplied via the keyboard.

Elementary checking of the input data is provided.

The output data describe the configuration (topology) of each solution and give parameters of the respective capacitors (in F) and inductors (in H).