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A Note on the Modeling of Transmission-Line Losses

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***Abstract*—We consider uniform lossy transmission lines characterized by their primary parameters. Exact and approximate formulas for the characteristic impedance and propagation coefficient are reviewed and discussed for low-loss lines. Approximating the characteristic impedance by its real part can lead to erroneous results for the input impedance of short-circuited and open-circuited stubs. This problem is analytically demonstrated on electrically short stubs. Results obtained using the exact and approximate expressions are compared with numerical solutions that are generated by various circuit simulation software.**

I. INTRODUCTION

In this paper, we consider the classical problem of transmission-line modeling by circuit-theory equations. This modeling is important because it often represents the first cut in the analysis and design of microwave circuits, and because it is sufficiently accurate for lower microwave frequencies. Electromagnetic simulation can be used for more accurate design to include fringe fields, parasitics, radiation, etc.

We assume the line to be uniform and in the sinusoidal regime at the angular frequency ω. The line is described by its primary parameters: L' – per-unit-length inductance, C' – per-unit-length capacitance, R' – per-unit-length resistance, and G' – per-unit-length conductance. The line length is D.

From the telegraphers' equations, one can derive the exact expressions for the characteristic impedance of the line,

$$Z_c = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}\,, \tag{1}$$

and the propagation coefficient,

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,. \tag{2}$$

For most microwave transmission lines, conductor and dielectric losses are relatively low. Hence, we may assume

$$R' << \omega L'\,, \tag{3}$$

$$G' << \omega C'\,. \tag{4}$$

Based on the above assumptions, the characteristic impedance is often approximated as purely real and given by

$$Z_c \approx \sqrt{\frac{L'}{C'}}\,. \tag{5}$$

This expression is commonly used in practice.

The propagation coefficient is usually properly taken to be complex, i.e., $\gamma = \alpha + j\beta$, where α is the attenuation coefficient and β is the phase coefficient. After expanding in a series, the propagation coefficient can be approximately evaluated as

$$\gamma \approx \frac{R'}{2\sqrt{\frac{L'}{C'}}} + \frac{G'}{2\sqrt{\frac{C'}{L'}}} + j\omega\sqrt{L'C'}\,, \tag{6}$$

where

$$\alpha = \frac{R'}{2\sqrt{\frac{L'}{C'}}} + \frac{G'}{2\sqrt{\frac{C'}{L'}}} \tag{7}$$

and

$$\beta = \omega\sqrt{L'C'} \,. \tag{8}$$

The attenuation coefficient can be represented as the sum of two terms, $\alpha = \alpha_{\mathrm{c}} + \alpha_{\mathrm{d}}$, where the first term,

$$\alpha_{\mathrm{c}} = \frac{R'}{2\sqrt{\frac{L'}{C'}}} \,, \tag{9}$$

is due to the conductor losses, and the second term,

$$\alpha_{\mathrm{d}} = \frac{G'}{2\sqrt{\frac{C'}{L'}}} \,, \tag{10}$$

is due to the dielectric losses.

The aim of this paper is to call attention to a problem that can be encountered when previous approximations are used in the analysis of short-circuited and open-circuited transmission-line sections (stubs). The approximation (5) leads to erroneous evaluation of transmission-line losses, which ultimately affect analysis results for the whole microwave network.

In Section II, this problem is analytically demonstrated on electrically short stubs. These results are numerically verified in Section III. In that section, we compare results obtained using the exact and approximate expressions from Sections I and II with numerical solutions generated by various circuit simulation software [1-6].

II. Short-Length Lossy Stubs

Let us consider an electrically short section of a uniform lossy transmission line. We shall analyze the input impedance of a short-circuited section and of an open-circuited section using the exact and approximate expressions for the characteristic impedance and the propagation coefficient.

A. Short-circuited stub

The exact expression for the input impedance of a short-circuited line is

$$Z_{\mathrm{ins}} = Z_{\mathrm{c}} \tanh(\gamma D) \,. \tag{11}$$

For a short line, $|\gamma D| << 1$, so we can approximate the hyperbolic tangent in (11) by

$$\tanh(\gamma D) \approx \gamma D \,. \tag{12}$$

Hence, $Z_{\mathrm{ins}} \approx Z_{\mathrm{c}}\gamma D$. Using the exact expressions (1) and (2), we further obtain

$$Z_{\mathrm{ins}} \approx \left(R' + \mathrm{j}\omega L'\right) D \,, \tag{13}$$

where $R'D$ is the total (series) resistance of the line, and $L'D$ is the total inductance. The result given by (13) is obvious for a microwave practitioner: the line approximately acts like an inductor whose quality factor is $Q_{\mathrm{L}} = \frac{\omega L'}{R'}$.

If we use (5) and (6) instead of (1) and (2), we obtain in a similar way from (11):

$$Z_{\mathrm{ins}} \approx \sqrt{\frac{L'}{C'}}\left(\alpha_{\mathrm{c}} + \alpha_{\mathrm{d}} + \mathrm{j}\omega\sqrt{L'C'}\right) D = \left(\frac{R'}{2} + \frac{G'L'}{2C'} + \mathrm{j}\omega L'\right) D \,. \tag{14}$$

In comparison with (13), equation (14) yields the same result for the imaginary part, but a different expression for the real part of the input impedance. Hence, the quality factors obtained from (13) and (14) are also different. For example, if the conductor losses dominate (i.e., if $\alpha_c >> \alpha_d$), (14) yields a two times smaller real part than (13), and, hence, a two times higher quality factor. If the dielectric losses dominate (i.e., if $\alpha_c << \alpha_d$), (14) gives overestimated losses, i.e., an underestimated quality factor. We shall demonstrate in Section III that equation (13) gives numerical results similar to (11). The different result obtained from (14) is primarily due to using (5) instead of (1).

B. Open-circuited stub

Now, let us consider a dual situation – an open-circuited transmission line. The exact input admittance to the line is

$$Y_{\mathrm{ino}} = Y_{\mathrm{c}} \tanh(\gamma D)\,, \tag{15}$$

where $Y_{\mathrm{c}} = 1/Z_{\mathrm{c}}$ is the characteristic admittance of the line. Using (1), (2), and (12), we obtain

$$Y_{\mathrm{ino}} \approx (G' + \mathrm{j}\omega C')D\,, \tag{16}$$

where $G'D$ is the total (parallel) conductance of the line, and $C'D$ is the total capacitance. This line acts like a capacitor whose quality factor is $Q_{\mathrm{C}} = \dfrac{\omega C'}{G'}$.

If we use (5) and (6) instead of (1) and (2), we obtain

$$Y_{\mathrm{ino}} \approx \sqrt{\frac{C'}{L'}}\left(\alpha_{\mathrm{c}} + \alpha_{\mathrm{d}} + \mathrm{j}\omega\sqrt{L'C'}\right)D = \left(\frac{R'C'}{2L'} + \frac{G'}{2} + \mathrm{j}\omega C'\right)D\,. \tag{17}$$

In comparison with (16), equation (17) yields the same imaginary part, but a different real part of the input admittance. If the conductor losses on the transmission line dominate (i.e., if $\alpha_c >> \alpha_d$), (17) gives overestimated losses. If the dielectric losses dominate (i.e., if $\alpha_c << \alpha_d$), (17) yields a two times better quality factor than (16).

III. Numerical Example

To investigate and illustrate the sources of discrepancy among various formulas of Section II, we consider a numerical example. It is a transmission line with the following primary parameters:

- $L' = 228.8\ \mathrm{nH/m}$,
- $C' = 91.54\ \mathrm{pF/m}$,
- $R' = 7.640\ \Omega/\mathrm{m}$,
- $G' = 485.2\ \mu\mathrm{S/m}$,

which are realistic data for a microstrip line at 1 GHz [7]. The line is assumed to be short-circuited and its length to be $D = 10$ mm.

As summarized in Table 1 (Case #1), the characteristic impedance of the line, according to (1), is $Z_{\mathrm{c}} = (49.99 - \mathrm{j}0.1118)\,\Omega$. The propagation coefficient, according to (2), is $\gamma = (0.088537 + \mathrm{j}28.755)\ \mathrm{m}^{-1}$. From γ, one can calculate the attenuation coefficient, $\alpha = 0.7690\ \mathrm{dB/m}$ (most of which is due to the conductor losses, because $\alpha_c = 0.6636\ \mathrm{dB/m}$) and evaluate the effective relative permittivity, $\varepsilon_{re} = 1.882$. Hence, the line length (D) is much shorter than the wavelength ($\lambda = 218$ mm).

Three approximations are introduced in Sections I and II, given by equations (5), (6), and (12), respectively. Each one can be used alone or in a combination with other approximations. To estimate the influence on the input impedance of the line, we introduce one approximation at a time. The results are given as Cases #2-4 in Table 1.

The approximation of the propagation coefficient, (6), has a negligible influence.

The approximation of the hyperbolic tangent, (13), introduces an error of about -6% for the real part of the input impedance and -3% for the imaginary part. This error diminishes for shorter line lengths.

The approximation of the characteristic impedance, (5), is critical. The resulting characteristic impedance is real and practically identical to the real part of (1). The difference is only in the small imaginary part, whose magnitude is about 0.2% of the real part. Although small, this number has the major impact on the real part of the input impedance. The resulting error is -41%, as predicted in Section II. In contrast, the approximation (5) has practically no influence on the imaginary part of the input impedance.

The combination of all three approximations, given by (14), gives practically the same imaginary part as (13). The real part given by (14) is significantly smaller, which, again, is attributed to using (5) instead of (1).

Table 1. Input impedance of a short-circuited transmission line.

Case	Formulas used	Z_c [Ω]	γ [m^{-1}]	Z_{ins} [Ω]
#1	Exact Z_c, γ, tanh (1), (2), (11)	49.99476 - j0.11176	0.0885368 + j28.7551	0.081187 + j14.78569
#2	Exact Z_c, γ Approximate tanh (13)	49.99476 - j0.11176	0.0885368 + j28.7551	0.076400 + j14.37593
#3	Exact Z_c, tanh Approximate γ (1), (6), (11)	49.99476 - j0.11176	0.0885370 + j28.7550	0.081188 + j14.78566
#4	Exact γ, tanh Approximate Z_c (5), (2), (11)	49.9945	0.0885368 + j28.7551	0.048135 + j14.78574
#5	Approximate Z_c, γ, tanh (14)	49.994537	0.0885370 + j28.7550	0.044264 + j14.37593
#6	Touchstone model			0.093660 + j14.78565

All microwave circuit simulators and many "low-frequency" simulators implement a transmission line as the circuit element. Usually, there are two models of the line. One is a lossless line, defined by its characteristic impedance and electrical length. The other model, referred to as the physical model, is usually given by the characteristic impedance that is assumed to be real, the attenuation coefficient, and the electrical length. We are interested here only in lossy lines. According to the results shown in Table 1, the data that define the physical model are insufficient to produce accurate results for the real part of the input impedance. This insufficiency has been often overlooked by both researchers and microwave software developers.

We have evaluated several available software packages. The programs of References [1-3] give identical results for the physical model of the transmission line as Case #4 in Table 1, i.e., they underestimate the real part of the input impedance.

The program of Reference [4] has a model that tries to bypass the problem of the real characteristic impedance. The model assumes all losses to be due to the conductor losses, i.e., it takes $G'=0$. It evaluates R' from the given attenuation coefficient and internally computes the characteristic impedance as a complex number. The resulting real part of the input impedance is larger than the exact one, as Case #6 in Table 1. The result would be correct only when the dielectric losses are negligibly small.

In certain programs, the input data for defining a lossy transmission line are all four primary parameters and the line length. This data set completely describes the line. One example is the program of Reference [5] that uses an exact formulation for the characteristic impedance and gives the same result as Case #1 in Table 1.

In the program of Reference [6], the standard lossy transmission-line model T-RLGC uses all four primary parameters. However, the result is the same as Case #4 in Table 1. On the other hand, the ABM module in this program, from the Library of transmission line models and subcircuits (tline.lib), gives correct results, as Case #1 in Table 1.

The same problem as investigated for the physical model of the transmission line was also discovered in other models in some programs (e.g., the coupled microstrip lines in [4]), but we shall not elaborate this further.

Figure 1 presents the relative error in the real part of the input impedance introduced by using (5) instead of (1) in (11), as a function of the line length. The error in the imaginary part is negligibly small, except in the immediate vicinity of resonant lengths. The error in the real part has a maximum for electrically short lines and it vanishes at resonant lengths. The envelope of this error diminishes with increasing the line length. A more detailed mathematical analysis of this error is beyond the scope of the paper.

For the same transmission line as above, the input admittance is calculated for an open-circuited stub. Equations (1), (2), and (15) give the exact input admittance, $Y_{\mathrm{ino}} = (6.0343 + \mathrm{j}5915.6)\,\mu\mathrm{S}$. Equations (16) and (17) give the input admittances $Y_{\mathrm{ino}} = (4.8520 + \mathrm{j}5751.6)\,\mu\mathrm{S}$, and $Y_{\mathrm{ino}} = (17.7093 + \mathrm{j}5751.6)\,\mu\mathrm{S}$, respectively. Equations (5), (2), and (15) yield $Y_{\mathrm{ino}} = (19.2580 + \mathrm{j}5915.6)\,\mu\mathrm{S}$. The last result, with a highly overestimated real part, is also obtained by the programs [1-3]. Touchstone yields $Y_{\mathrm{ino}} = (1.0441 + \mathrm{j}5915.6)\,\mu\mathrm{S}$. The real part is highly underestimated because the loss is associated only with conductors, whereas the major losses for this short line come from the dielectric (whose quality factor is $Q_{\mathrm{C}} = 1185$).

IV. CONCLUSION

We consider uniform lossy transmission lines characterized by the primary (per-unit-length) parameters at a given frequency. The lines are assumed to operate in the sinusoidal regime. We present exact and approximate formulas for the characteristic impedance and the propagation coefficient of low-loss lines. The usual approximation of the complex characteristic impedance by its real part introduces a significant error in the real part of the input impedance of transmission-line stubs. Consequently, the quality factor of the stubs can be overestimated or underestimated. This error is particularly pronounced for electrically short lines. A similar problem occurs in the time-domain analysis of lossy transmission lines of arbitrary lengths. The remedy is to adequately take into account all four primary parameters of the line.

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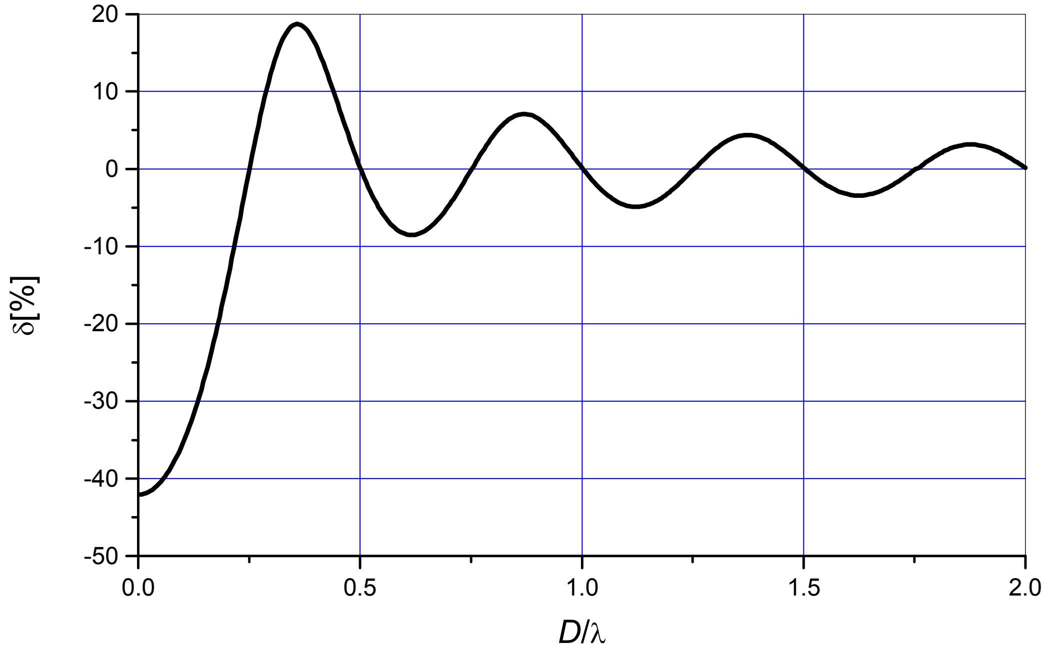

Figure 1. The relative error in the real part of the input impedance (δ) introduced by using (5) instead of (1) in (11), as a function of the normalized line length (D/λ).