

Comprehensive Table of Singular Moduli

(Collected and checked by V.V.Petrović)

([http://mtt.etf.bg.ac.rs/Singular Moduli](http://mtt.etf.bg.ac.rs/Singular_Moduli) [www/SingularModuliVVP.pdf](http://www.SingularModuliVVP.pdf))

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Contains values of 124 singular moduli

1 What are Singular Moduli

Consider the function $f(k) = \frac{K(k)}{K'(k)}$, $k \in [0, 1)$, where $K(k)$ is the complete elliptic integral of the first kind, $K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \theta)^{-\frac{1}{2}} d\theta$, k is its *elliptic modulus*, and $K'(k) = K(\sqrt{1 - k^2})$. Singular moduli k_n are elliptic moduli for which

$$f(k_n) = \frac{1}{\sqrt{n}} \quad (n = 1, 2, 3, \dots).$$

For brevity, many singular moduli in this table are given indirectly by the so-called *class invariants* g_n and G_n , from which k_n is obtained as

$$k_n = \frac{1}{g_n^{12} + \sqrt{1 + g_n^{24}}} \quad \text{and} \quad k_n = \frac{G_n^{-12}}{\sqrt{2}\sqrt{1 + \sqrt{1 - G_n^{-24}}}}.$$

2 Extension of the table

Due to elementary property of the function $f(k)$, for every k_n (except for k_1), another pair of explicit values of k and $f(k)$ can be obtained as

$$f(\sqrt{1 - k_n^2}) = \sqrt{n}.$$

3 The Table

k_1	$\frac{1}{\sqrt{2}}$
k_2	$-1 + \sqrt{2}$
k_3	$\frac{-1 + \sqrt{3}}{2\sqrt{2}}$
k_4	$3 - 2\sqrt{2}$
k_5	$\sqrt{\frac{1}{2} - \sqrt{-2 + \sqrt{5}}}$
k_6	$-3 - 2\sqrt{2} + 2\sqrt{3} + \sqrt{6}$
k_7	$\frac{1}{4}\sqrt{8 - 3\sqrt{7}}$
k_8	$5 + 4\sqrt{2} - 2\sqrt{2(7 + 5\sqrt{2})}$

k_9	$\frac{1}{\sqrt{\sqrt{194 + 112\sqrt{3} + 4\sqrt{4680 + 2702\sqrt{3}}}}}$
k_{10}	$-9 + 3\sqrt{10} - 2\sqrt{38 - 12\sqrt{10}}$
k_{11}	$\frac{1}{2} \sqrt{\frac{32 \cdot 3^{2/3} + 3\sqrt[3]{9 + 7\sqrt{33}} - 4\sqrt[3]{3} \left(9 + 7\sqrt{33}\right)^{2/3}}{6\sqrt[3]{9 + 7\sqrt{33}} + \sqrt{3 \left(-32 \cdot 3^{2/3} \sqrt[3]{9 + 7\sqrt{33}} + 9 \left(9 + 7\sqrt{33}\right)^{2/3} + 4\sqrt[3]{3} \left(9 + 7\sqrt{33}\right)\right)}}$
k_{12}	$(-3 + 2\sqrt{2})(-5 + 2\sqrt{6})$
k_{13}	$\frac{1}{\sqrt{2 \left(649 + 180\sqrt{13} + 6\sqrt{23382 + 6485\sqrt{13}}\right)}}$
g_{14}	$\sqrt{\frac{1}{2} \left(1 + \sqrt{2} + \sqrt{-1 + 2\sqrt{2}}\right)}$
k_{15}	$\frac{1}{2\sqrt{2 \left(376 + 168\sqrt{5} + \sqrt{6 \left(47067 + 21049\sqrt{5}\right)}\right)}}$
k_{16}	$(-1 + \sqrt[4]{2})^4 (3 + 2\sqrt{2})$
k_{17}	$\frac{1}{\sqrt{\frac{2 + 4\sqrt{-412 - 100\sqrt{17}} + 5\sqrt{13598 + 3298\sqrt{17}}}{1649 + 400\sqrt{17} - 20\sqrt{13598 + 3298\sqrt{17}}}}}$
k_{18}	$\sqrt{-56\sqrt{58803} - 41580\sqrt{2} + 97(99 - 70\sqrt{2})}$
k_{19}	$\frac{1}{2} \sqrt{\frac{96 + \sqrt[3]{1 + 3\sqrt{57}} - 12 \left(1 + 3\sqrt{57}\right)^{2/3}}{2\sqrt[3]{1 + 3\sqrt{57}} + \sqrt{3 \left(4 + 12\sqrt{57} - 32\sqrt[3]{1 + 3\sqrt{57}} + \left(1 + 3\sqrt{57}\right)^{2/3}\right)}}$
k_{21}	$-\frac{\left(-3 + \sqrt{7}\right) \left(-\sqrt{3} + \sqrt{7}\right)^{3/2} \left(\sqrt{-1 + \sqrt{7}} - \sqrt{3 + \sqrt{7}}\right)^2 \left(\sqrt{1 + \sqrt{7}} - \sqrt{5 + \sqrt{7}}\right)^2}{64\sqrt{2}}$
k_{22}	$(10 - 3\sqrt{11})(-7\sqrt{2} + 3\sqrt{11})$
G_{23}	$\frac{\sqrt[3]{9 - \sqrt{69}} + \sqrt[3]{9 + \sqrt{69}}}{\sqrt[12]{2} \cdot 3^{2/3}}$
k_{24}	$-\left(3 + 2\sqrt{3} - 2\sqrt{5 + 3\sqrt{3}}\right) \left(-11 - 6\sqrt{3} + 2\sqrt{57 + 33\sqrt{3}}\right)$
k_{25}	$\sqrt{\frac{1}{2} - 6\sqrt{-360 + 161\sqrt{5}}}$
g_{26}	$\frac{1}{3} \left(\sqrt{2 + \sqrt{13}} - \sqrt[3]{-(2 + \sqrt{13})^{3/2} + 3\sqrt{3}(3 + \sqrt{13})} + \sqrt[3]{(2 + \sqrt{13})^{3/2} + 3\sqrt{3}(3 + \sqrt{13})}\right)$
G_{27}	$\frac{\sqrt[12]{2}}{\sqrt[3]{-1 + \sqrt[3]{2}}}$
k_{28}	$(-1 + \sqrt{2})^4 (15 - 4\sqrt{14})$
k_{30}	$-(-2 + \sqrt{3})(-\sqrt{5} + \sqrt{6})(-5 + 2\sqrt{6})(-4 + \sqrt{15})$

G_{31}	$\frac{2 + 2^{2/3} \sqrt[3]{29 - 3\sqrt{93}} + 2^{2/3} \sqrt[3]{29 + 3\sqrt{93}}}{3 \cdot 2^{3/4}}$
k_{33}	$\frac{1}{32} (2 - \sqrt{3})^{3/2} (-3 + \sqrt{11}) \left(\sqrt{1 + \sqrt{3}} - \sqrt{5 + \sqrt{3}} \right)^2 \left(\sqrt{3(1 + \sqrt{3})} - \sqrt{7 + 3\sqrt{3}} \right)^2$
g_{34}	$\frac{1}{2} \sqrt{3 + \sqrt{17} + \sqrt{10 + 6\sqrt{17}}}$
G_{37}	$\sqrt[4]{6 + \sqrt{37}}$
g_{38}	$\frac{4\sqrt[3]{2} - 2 \cdot 2^{5/6} + 2\sqrt{2} (1 + \sqrt{2})^{2/3} \sqrt[3]{17 + 4\sqrt{2} + 3\sqrt{57}} + \sqrt[3]{1 + \sqrt{2}} (34 + 8\sqrt{2} + 6\sqrt{57})^{2/3}}{6\sqrt[6]{1 + \sqrt{2}} \sqrt[3]{17 + 4\sqrt{2} + 3\sqrt{57}}}$
G_{39}	$\frac{\sqrt[6]{3 + \sqrt{13}} \left(\sqrt{2(-3 + \sqrt{13})} + \sqrt{2(5 + \sqrt{13})} \right)}{2 \cdot 2^{11/12}}$
k_{40}	$\left(2\sqrt{2} + \sqrt{5} - 2\sqrt{3 + \sqrt{10}} \right)^2 \left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})} \right)^2$
G_{41}	$\frac{1}{2} \sqrt{\frac{1}{2} \left(5 + \sqrt{41} + \sqrt{2(5 + \sqrt{41})} + 2\sqrt{\frac{138 + 18\sqrt{41} + 33\sqrt{2(5 + \sqrt{41})} + 5\sqrt{82(5 + \sqrt{41})}}{5 + \sqrt{41}}} \right)}$
k_{42}	$(-3 + 2\sqrt{2}) (-7 + 4\sqrt{3}) (\sqrt{6} - \sqrt{7}) (-8 + 3\sqrt{7})$
G_{43}	$\frac{2 + \sqrt[3]{35 - 3\sqrt{129}} + \sqrt[3]{35 + 3\sqrt{129}}}{3\sqrt[4]{2}}$
k_{45}	$-\frac{(-2 + \sqrt{5})^{3/2} (-4 + \sqrt{15}) \left(\sqrt{1 + \sqrt{5}} - \sqrt{3 + \sqrt{5}} \right)^2 \left(\sqrt{3(1 + \sqrt{5})} - \sqrt{7 + 3\sqrt{5}} \right)^2}{8\sqrt{2}}$
g_{46}	$\sqrt{\frac{1}{2} (3 + \sqrt{2} + \sqrt{7 + 6\sqrt{2}})}$
G_{49}	$\frac{1}{2} \left(\sqrt[4]{7} + \sqrt{4 + \sqrt{7}} \right)$
g_{50}	$\frac{1}{6} \left(2 + \sqrt[3]{2(5 + \sqrt{5}) (1 + 7\sqrt{5} - 6\sqrt{6})} + \sqrt[3]{2(5 + \sqrt{5}) (1 + 7\sqrt{5} + 6\sqrt{6})} \right)$
k_{55}	$\frac{(-2 + \sqrt{5}) \sqrt{66 + 30\sqrt{5} - 20\sqrt{11} - 9\sqrt{55}} \left(\sqrt{2(-1 + \sqrt{5})} - \sqrt{2(7 + \sqrt{5})} \right)^6 (3 + \sqrt{5} - \sqrt{13 + 6\sqrt{5}})}{2^{14}}$
G_{57}	$\frac{\sqrt[4]{2 + \sqrt{3}} \sqrt[6]{13 + 3\sqrt{19}}}{\sqrt[12]{2}}$
k_{58}	$(99 - 70\sqrt{2}) (-99 + 13\sqrt{58})$
k_{60}	$(3 - 2\sqrt{2}) (5 - 2\sqrt{6}) (-3 + \sqrt{10})^2 (11 - 2\sqrt{30})$
g_{62}	$\frac{1}{4} \left(\sqrt{1 + \sqrt{2}} + \sqrt{9 + 5\sqrt{2}} + \sqrt{2(-3 + 3\sqrt{2} + \sqrt{19 + 14\sqrt{2}})} \right)$
G_{63}	$\frac{1}{4} \sqrt[6]{5 + \sqrt{21}} \sqrt[12]{2} \left(\sqrt{2(-3 + \sqrt{21})} + \sqrt{2(5 + \sqrt{21})} \right)$
G_{65}	$\frac{1}{2} \sqrt[4]{\frac{1}{2} (1 + \sqrt{5}) (3 + \sqrt{13})} \sqrt{\sqrt{1 + \sqrt{65}} + \sqrt{9 + \sqrt{65}}}$
g_{66}	$\frac{1}{2} \sqrt[4]{\sqrt{2} + \sqrt{3}} \sqrt[12]{7\sqrt{2} + 3\sqrt{11}} \sqrt{\sqrt{2(-1 + \sqrt{33})} + \sqrt{2(7 + \sqrt{33})}}$

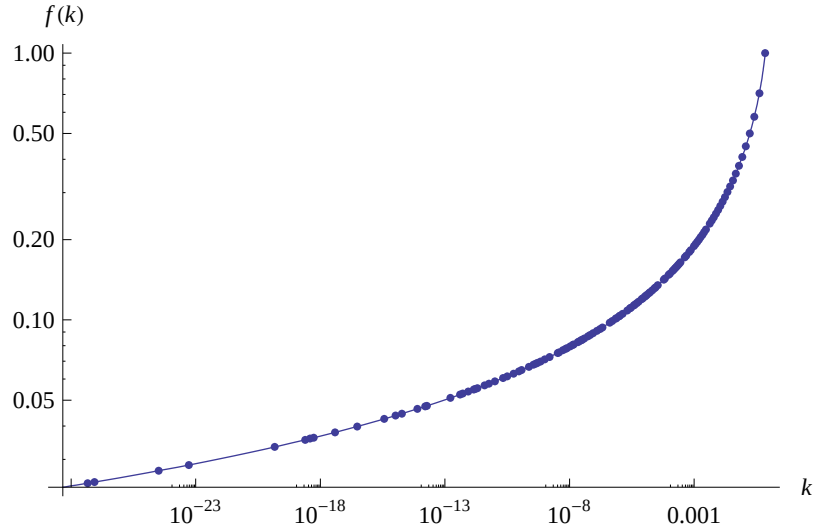
G_{67}	$\frac{2 + \sqrt[3]{53 - 3\sqrt{201}} + \sqrt[3]{53 + 3\sqrt{201}}}{3\sqrt[4]{2}}$
G_{69}	$\frac{\sqrt[12]{5 + \sqrt{23}} \sqrt[8]{3\sqrt{3} + \sqrt{23}} \sqrt{\sqrt{3}(2 + \sqrt{3}) + \sqrt{2 + 3\sqrt{3}}}}{2^{2/3}}$
k_{70}	$(8 - 3\sqrt{7})(-5\sqrt{5} + 3\sqrt{14})(-15 + 4\sqrt{14})(-6 + \sqrt{35})$
G_{73}	$\frac{1}{4} \left(\sqrt{2(1 + \sqrt{73})} + \sqrt{2(9 + \sqrt{73})} \right)$
G_{75}	$\frac{6 \cdot 2^{5/12}}{-2 - 2^{2/3} \sqrt[6]{5} - 2\sqrt{5} + 5^{5/6} \sqrt[3]{2} + \sqrt[3]{10} + 10^{2/3}}$
G_{77}	$\frac{\sqrt[8]{21 + 8\sqrt{7} + 8\sqrt{11} + 3\sqrt{77}} \sqrt{\sqrt{2 + \sqrt{11}} + \sqrt{6 + \sqrt{11}}}}{2^{5/8}}$
k_{78}	$(-2 + \sqrt{3})^3 (-5 + 2\sqrt{6})(3\sqrt{3} - \sqrt{26})(25 - 4\sqrt{39})$
G_{81}	$\sqrt[3]{\frac{1 + \sqrt[3]{2(1 + \sqrt{3})}}{-1 + \sqrt[3]{2(-1 + \sqrt{3})}}}$
g_{82}	$\frac{1}{4} \left(\sqrt{2(5 + \sqrt{41})} + \sqrt{2(13 + \sqrt{41})} \right)$
G_{85}	$\frac{1}{2} (1 + \sqrt{5}) \sqrt[4]{\frac{1}{2} (9 + \sqrt{85})}$
g_{90}	$\frac{1}{2} \sqrt[6]{5 + 2\sqrt{5} + 2\sqrt{6} + \sqrt{30}} \left(\sqrt{-1 + \sqrt{6}} + \sqrt{3 + \sqrt{6}} \right)$
G_{93}	$\frac{\sqrt[4]{3\sqrt{3} + \sqrt{31}} \sqrt[6]{39 + 7\sqrt{31}}}{\sqrt[3]{2}}$
g_{94}	$\frac{1}{4} \left(\sqrt{7 + \sqrt{2}} + \sqrt{7 + 5\sqrt{2}} + \sqrt{2(-1 + 3\sqrt{2} + \sqrt{59 + 42\sqrt{2}})} \right)$
G_{97}	$\frac{1}{4} \left(\sqrt{2(5 + \sqrt{97})} + \sqrt{2(13 + \sqrt{97})} \right)$
g_{98}	$\frac{1}{4} \left(\sqrt{2} + \sqrt{14 + 4\sqrt{14}} + 2\sqrt{\sqrt{14} + \sqrt{7 + 2\sqrt{14}}} \right)$
k_{102}	$\frac{1}{2} (-7 + 4\sqrt{3})(-49 + 20\sqrt{6})(-7 + \sqrt{51})^2 (-5\sqrt{2} + \sqrt{51})$
G_{105}	$\sqrt[6]{2 + \sqrt{5}} \sqrt[4]{\frac{1}{2} (2 + \sqrt{3})} (5 + \sqrt{21}) \sqrt[12]{6 + \sqrt{35}}$
g_{114}	$\frac{1}{2} \sqrt[4]{\sqrt{2} + \sqrt{3}} \sqrt[12]{3\sqrt{2} + \sqrt{19}} \sqrt{\sqrt{6(5 + \sqrt{57})} + \sqrt{46 + 6\sqrt{57}}}$
G_{117}	$\frac{\sqrt[4]{3 + \sqrt{13}} \sqrt[6]{2\sqrt{3} + \sqrt{13}} \left(\sqrt[4]{3} + \sqrt{4 + \sqrt{3}} \right)}{2\sqrt[4]{2}}$
G_{121}	$\frac{2 + \sqrt[3]{11 + 3\sqrt{11}} \left(\sqrt[3]{4 - 3\sqrt{3} + 3\sqrt{11}} + \sqrt[3]{4 + 3\sqrt{3} + 3\sqrt{11}} \right)}{3\sqrt{2}}$
g_{126}	$\frac{\sqrt{\sqrt{3} + \sqrt{7}} \sqrt[6]{\sqrt{6} + \sqrt{7}} \left(1 + \sqrt{2} + \sqrt{-1 + 2\sqrt{2}} \right)}{2\sqrt{2}}$
k_{130}	$(17 - 12\sqrt{2})(-3 + \sqrt{10})^2 (-5 + \sqrt{26})^2 (-57 + 5\sqrt{130})$
G_{133}	$\sqrt[4]{\frac{1}{2} (8 + 3\sqrt{7})} (5\sqrt{7} + 3\sqrt{19})$
g_{138}	$\frac{\sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[12]{78\sqrt{2} + 23\sqrt{23}} \sqrt{\sqrt{1 + 2\sqrt{6}} + \sqrt{5 + 2\sqrt{6}}}}{2^{3/4}}$

G_{141}	$\frac{\sqrt[12]{7 + \sqrt{47}} \sqrt[8]{4\sqrt{3} + \sqrt{47}} \sqrt[3]{3\sqrt{2 + \sqrt{3}} + \sqrt{14 + 9\sqrt{3}}}}{\sqrt{2} \sqrt[24]{2}}$
g_{142}	$\sqrt{\frac{1}{2} \left(9 + 5\sqrt{2} + \sqrt{127 + 90\sqrt{2}} \right)}$
G_{145}	$\frac{1}{2} \sqrt[4]{\frac{1}{2} (2 + \sqrt{5}) (5 + \sqrt{29})} \sqrt{\sqrt{2 (9 + \sqrt{145})} + \sqrt{2 (17 + \sqrt{145})}}$
G_{147}	$\frac{6 \sqrt[12]{2}}{3 - 2 \sqrt[3]{2} \sqrt[3]{3} \sqrt[6]{7} + \sqrt{21}}$
G_{153}	$\frac{1}{4} \left(1 + \sqrt{17} + \sqrt{2 (1 + \sqrt{17})} \right) \sqrt[3]{\frac{1}{2} \left(\sqrt{33 + 9\sqrt{17}} + \sqrt{37 + 9\sqrt{17}} \right)}$
g_{154}	$\frac{\sqrt[4]{7 + 2\sqrt{14} + 2\sqrt{22} + \sqrt{77}} \sqrt{\sqrt{9 + 2\sqrt{22}} + \sqrt{13 + 2\sqrt{22}}}}{2^{3/4}}$
g_{158}	$\frac{1}{4} \left(\sqrt{9 + \sqrt{2}} + \sqrt{17 + 13\sqrt{2}} + \sqrt{2 \left(5 + 7\sqrt{2} + \sqrt{179 + 134\sqrt{2}} \right)} \right)$
G_{163}	$\frac{6 + \sqrt[3]{135 - 3\sqrt{489}} + \sqrt[3]{3 \left(45 + \sqrt{489} \right)}}{3 \sqrt[4]{2}}$
G_{165}	$\frac{\sqrt[6]{(2 + \sqrt{5}) (3\sqrt{5} + 2\sqrt{11})} \sqrt[4]{(4 + \sqrt{15}) (\sqrt{11} + \sqrt{15})}}{\sqrt[4]{2}}$
G_{169}	$\frac{1}{6} \left(4 + 2\sqrt{13} + \sqrt[3]{2 (11 - 6\sqrt{3} + \sqrt{13}) (13 + 3\sqrt{13})} + \sqrt[3]{2 (11 + 6\sqrt{3} + \sqrt{13}) (13 + 3\sqrt{13})} \right)$
G_{175}	$\frac{6 \sqrt[4]{2}}{-1 + \sqrt{5} + \sqrt[3]{2 (5 - \sqrt{5}) (8 - 3\sqrt{5} + 3\sqrt{21})} - \sqrt[3]{2 (5 - \sqrt{5}) (-8 + 3\sqrt{5} + 3\sqrt{21})}}$
G_{177}	$\frac{(1 + \sqrt{3})^{3/2} \sqrt[6]{23 + 3\sqrt{59}}}{2^{5/6}}$
k_{190}	$(51\sqrt{10} - 37\sqrt{19}) (-170 + 39\sqrt{19}) (-37 + 6\sqrt{38}) (-39 + 4\sqrt{95})$
g_{198}	$\frac{1}{4} \sqrt{1 + \sqrt{2}} \sqrt[6]{4\sqrt{2} + \sqrt{33}} \left(\sqrt{2 (1 + \sqrt{33})} + \sqrt{2 (9 + \sqrt{33})} \right)$
G_{205}	$\frac{1}{8} (1 + \sqrt{5}) \sqrt[4]{\frac{1}{2} (3\sqrt{5} + \sqrt{41})} \left(\sqrt{2 (-1 + \sqrt{41})} + \sqrt{2 (7 + \sqrt{41})} \right)$
k_{210}	$(-3 + 2\sqrt{2}) (-2 + \sqrt{3}) (-8 + 3\sqrt{7}) (-3 + \sqrt{10})^2 (\sqrt{14} - \sqrt{15}) (-4 + \sqrt{15})^2 (-6 + \sqrt{35}) (-13 + 2\sqrt{42})$
G_{213}	$\frac{\sqrt[8]{5\sqrt{3} + \sqrt{71}} \sqrt[12]{59 + 7\sqrt{71}} \sqrt{\sqrt{6 (7 + 4\sqrt{3})} + \sqrt{38 + 24\sqrt{3}}}}{2^{2/3}}$
G_{217}	$\frac{1}{2} \sqrt{\left(\sqrt{12 + 5\sqrt{7}} + \sqrt{16 + 5\sqrt{7}} \right) \left(\sqrt{18 + 8\sqrt{7}} + \sqrt{22 + 8\sqrt{7}} \right)}$
G_{225}	$\frac{1}{4} \sqrt[3]{2 + \sqrt{3}} (1 + \sqrt{5}) \left(\sqrt[4]{15} + \sqrt{4 + \sqrt{15}} \right)$
g_{238}	$\frac{1}{4} \left(\sqrt{1 + 2\sqrt{2}} + \sqrt{5 + 2\sqrt{2}} \right) \left(\sqrt{1 + 3\sqrt{2}} + \sqrt{5 + 3\sqrt{2}} \right)$
G_{243}	$\frac{\sqrt[12]{2} \left(\sqrt[3]{2} + 2^{2/3} + \sqrt[3]{3} \right)}{\sqrt[3]{9 - 6\sqrt{3}}}$
G_{253}	$\sqrt{\frac{1}{2} (5 + \sqrt{23}) \sqrt[4]{13\sqrt{11} + 9\sqrt{23}}}$
G_{265}	$\frac{1}{2} \sqrt[4]{\frac{1}{2} (2 + \sqrt{5}) (7 + \sqrt{53})} \sqrt{\sqrt{2 (81 + 5\sqrt{265})} + \sqrt{2 (89 + 5\sqrt{265})}}$

G_{273}	$\frac{\sqrt[4]{2 + \sqrt{3}} \sqrt{(\sqrt{3} + \sqrt{7}) (3 + \sqrt{13})} \sqrt[6]{15\sqrt{7} + 11\sqrt{13}}}{2^{\frac{12}{\sqrt{2}}}}$
G_{289}	$\frac{1}{16} \left(\sqrt{1 + 5\sqrt[4]{17}} + \sqrt{17 + 17^{3/4}} + \sqrt{17 + 5\sqrt[4]{17}} + \sqrt{17 + 17^{3/4}} \right)^2$
G_{301}	$\frac{\sqrt[8]{1197 + 456\sqrt{7} + 184\sqrt{43}} + 69\sqrt{301} \sqrt{\sqrt{7(6 + \sqrt{43})} + \sqrt{46 + 7\sqrt{43}}}}{2^{5/8}}$
g_{310}	$\frac{1}{4} \sqrt{1 + \sqrt{2}} (1 + \sqrt{5}) \left(\sqrt{3 + 2\sqrt{10}} + \sqrt{7 + 2\sqrt{10}} \right)$
G_{325}	$\frac{\sqrt[4]{\frac{1}{2} (3 + \sqrt{13})}}{6 (-1 + \sqrt{5})} \left(7 + \sqrt{5} - \sqrt{13} + \sqrt{65} + \frac{2^{2/3} (5 + 3\sqrt{5}) (3 + \sqrt{13})}{\sqrt[3]{25 + 71\sqrt{5} + 35\sqrt{13} + 5\sqrt{65} + 6\sqrt{15(27 - 9\sqrt{5} - 5\sqrt{13} + 3\sqrt{65})}}} \right. \\ \left. + 2\sqrt[3]{2(25 + 71\sqrt{5} + 35\sqrt{13} + 5\sqrt{65} + 6\sqrt{15(27 - 9\sqrt{5} - 5\sqrt{13} + 3\sqrt{65})})} \right)$
g_{330}	$\frac{1}{2} (1 + \sqrt{5}) \sqrt[6]{\sqrt{10} + \sqrt{11}} \sqrt{\frac{1}{2} (\sqrt{5} + \sqrt{6}) (\sqrt{11} + \sqrt{15})}$
G_{333}	$\frac{1}{2} \sqrt[4]{6 + \sqrt{37}} \sqrt[6]{7\sqrt{3} + 2\sqrt{37}} \left(\sqrt{3 + 2\sqrt{3}} + \sqrt{7 + 2\sqrt{3}} \right) \\ \frac{(1 + \sqrt{5}) \sqrt[6]{15\sqrt{5} + 7\sqrt{23}} \sqrt{9 + 3\sqrt{3} + \sqrt{23}} + \sqrt{69}}{2 \cdot 2^{5/6}}$
G_{345}	$\frac{(\sqrt{3} + \sqrt{7})^4 \sqrt{(8 + 3\sqrt{7}) (\sqrt{17} + \sqrt{21})} \sqrt[6]{11 + \sqrt{119}}}{2^{3/2}}$
G_{357}	$\frac{1}{3 \cdot 2^{7/12}} \left(4 + \sqrt{33} + \sqrt{11 + 2\sqrt{33}} + \frac{2 \left(33 + 5\sqrt{33} + 7\sqrt{11 + 2\sqrt{33}} + \sqrt{363 + 66\sqrt{33}} \right)}{\sqrt[3]{979 + 165\sqrt{33} + 203\sqrt{11 + 2\sqrt{33}} + 35\sqrt{363 + 66\sqrt{33}} + 6\sqrt{3(429 + 74\sqrt{33} + 88\sqrt{11 + 2\sqrt{33}} + 16\sqrt{363 + 66\sqrt{33}})}}} \right. \\ \left. + \sqrt[3]{979 + 165\sqrt{33} + 203\sqrt{11 + 2\sqrt{33}} + 35\sqrt{363 + 66\sqrt{33}} + 6\sqrt{3(429 + 74\sqrt{33} + 88\sqrt{11 + 2\sqrt{33}} + 16\sqrt{363 + 66\sqrt{33}})}} \right)$
G_{385}	$\frac{1}{2} \sqrt{\frac{1}{2} (3 + \sqrt{5}) (\sqrt{5} + \sqrt{7}) (3 + \sqrt{11}) (\sqrt{7} + \sqrt{11})}$

G_{441}	$\frac{1}{2} \sqrt[6]{2 + \sqrt{3}} \sqrt{\frac{(\sqrt{3} + \sqrt{7}) \left(\sqrt[4]{6} \sqrt[8]{7} + \sqrt{3 + \sqrt{7}} \right) \left(2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{-\sqrt[4]{6} \sqrt[8]{7} + \sqrt{3 + \sqrt{7}}}}$
G_{445}	$\frac{1}{4} \sqrt{2 + \sqrt{5}} \sqrt[4]{\frac{1}{2} (21 + \sqrt{445})} \left(\sqrt{2 (5 + \sqrt{89})} + \sqrt{2 (13 + \sqrt{89})} \right)$
G_{465}	$\frac{1}{4} \sqrt{2} \sqrt[4]{(2 + \sqrt{3}) (1 + \sqrt{5}) (3\sqrt{3} + \sqrt{31})} \sqrt[12]{5\sqrt{5} + 2\sqrt{31}} \\ \times \sqrt{\left(\sqrt{2 + \sqrt{31}} + \sqrt{6 + \sqrt{31}} \right) \left(\sqrt{22 + 4\sqrt{31}} + \sqrt{26 + 4\sqrt{31}} \right)}$
G_{505}	$\frac{1}{2} \sqrt[4]{\frac{1}{2} (1 + \sqrt{5}) (10 + \sqrt{101})} \sqrt{(2 + \sqrt{5}) \left(\sqrt{10 (21 + \sqrt{505})} + \sqrt{226 + 10\sqrt{505}} \right)}$
g_{522}	$\frac{1}{4} \sqrt{2} \sqrt{5 + \sqrt{29}} \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \left(\sqrt{3 (3 + \sqrt{6})} + \sqrt{5 + 3\sqrt{6}} \right)$
G_{553}	$\frac{1}{2} \sqrt{\left(\sqrt{96 + 11\sqrt{79}} + \sqrt{100 + 11\sqrt{79}} \right) \left(\sqrt{282 + 32\sqrt{79}} + \sqrt{286 + 32\sqrt{79}} \right)}$
g_{630}	$\frac{1}{16} \sqrt{(1 + \sqrt{2}) (3 + \sqrt{5}) (\sqrt{3} + \sqrt{7})} \sqrt[6]{\sqrt{14} + \sqrt{15}} \left(\sqrt{-2 + \sqrt{7} + \sqrt{15}} + \sqrt{2 + \sqrt{7} + \sqrt{15}} \right) \\ \times \left(\sqrt{2 (-4 + \sqrt{7} + \sqrt{15})} + \sqrt{2 (4 + \sqrt{7} + \sqrt{15})} \right)$
G_{697}	$\frac{1}{4} \sqrt{\left(\sqrt{1306 + 50\sqrt{697}} + \sqrt{1322 + 50\sqrt{697}} \right) \left(\sqrt{1522 + 58\sqrt{697}} + \sqrt{1538 + 58\sqrt{697}} \right)}$
G_{765}	$\frac{1}{4 \cdot 2^{7/12}} (1 + \sqrt{5}) \sqrt[6]{\sqrt{15} + \sqrt{17}} \sqrt[4]{9 + \sqrt{85}} \sqrt{\sqrt{3 (6 + \sqrt{51})} + \sqrt{22 + 3\sqrt{51}}} \\ \times \sqrt{(\sqrt{3} + \sqrt{5}) \left(\sqrt{6 + \sqrt{51}} + \sqrt{10 + \sqrt{51}} \right)}$
G_{777}	$\frac{1}{2} \sqrt[4]{\frac{1}{2} (3 + 2\sqrt{3} + 2\sqrt{7} + \sqrt{21}) (6 + \sqrt{37})} \sqrt[12]{246\sqrt{7} + 107\sqrt{37}} \\ \times \sqrt{\left(\sqrt{3 (2 + \sqrt{7})} + \sqrt{10 + 3\sqrt{7}} \right) \left(\sqrt{6 (5 + 2\sqrt{7})} + \sqrt{34 + 12\sqrt{7}} \right)}$
G_{793}	$\sqrt{\frac{1}{2} \left(2\sqrt{28 + \sqrt{793}} + \sqrt{113 + 4\sqrt{793}} \right) \left(5\sqrt{28 + \sqrt{793}} + \sqrt{704 + 25\sqrt{793}} \right)}$
G_{897}	$\frac{1}{4} \sqrt[4]{2} \sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[12]{4\sqrt{13} + 3\sqrt{23}} \sqrt{\sqrt{56 + 9\sqrt{39}} + \sqrt{60 + 9\sqrt{39}}} \\ \times \sqrt{(2 + \sqrt{3}) (3 + \sqrt{13}) \left(\sqrt{4 + \sqrt{39}} + \sqrt{8 + \sqrt{39}} \right)}$
G_{1225}	$\frac{1}{32} (1 + \sqrt{5}) \sqrt{\sqrt{10} + \sqrt{14}} \left(\sqrt[4]{7} + \sqrt{4 + \sqrt{7}} \right)^{3/2} \\ \times \left(\sqrt{70 + 30\sqrt{7}} + 16\sqrt[4]{7}\sqrt{10} + 6 \cdot 7^{3/4}\sqrt{10} + \sqrt{86 + 30\sqrt{7}} + 16\sqrt[4]{7}\sqrt{10} + 6 \cdot 7^{3/4}\sqrt{10} \right)$
G_{1353}	$\frac{1}{4 \cdot 2^{5/12}} \sqrt[4]{(5 + 3\sqrt{3}) (3 + \sqrt{11}) (11 + \sqrt{123})} \sqrt[12]{6817 + 321\sqrt{451}} \\ \times \sqrt{\left(\sqrt{34 + 6\sqrt{33}} + \sqrt{50 + 6\sqrt{33}} \right) \left(\sqrt{66 (17 + 3\sqrt{33})} + \sqrt{2 (569 + 99\sqrt{33})} \right)}$
G_{1645}	$\frac{1}{4} \sqrt[4]{(3 + \sqrt{7}) (7 + \sqrt{47})} \sqrt[8]{\frac{1}{2} (73\sqrt{5} + 9\sqrt{329})} \\ \times \sqrt{(2 + \sqrt{5}) \left(\sqrt{7 (17 + \sqrt{329})} + \sqrt{127 + 7\sqrt{329}} \right) \left(\sqrt{1486 + 82\sqrt{329}} + \sqrt{1502 + 82\sqrt{329}} \right)}$
G_{1677}	$\frac{1}{4} (3 + \sqrt{13})^{3/4} \sqrt[12]{4414\sqrt{13} + 2427\sqrt{43}} \sqrt[4]{13\sqrt{3} + 6\sqrt{13} + 2\sqrt{129} + \sqrt{559}} \\ \times \sqrt{\left(\sqrt{13 + 2\sqrt{43}} + \sqrt{17 + 2\sqrt{43}} \right) \left(3\sqrt{39 + 6\sqrt{43}} + \sqrt{355 + 54\sqrt{43}} \right)}$

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Graphical representation of the singular moduli listed in the Table.

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